Supersymmetric extension of the standard model with naturally stable proton

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A new supersymmetric standard model based on $N=1$ supergravity is constructed, aiming at a natural explanation of proton stability without invoking an *ad hoc* discrete symmetry through R parity. The proton is protected from decay by an extra U(1) gauge symmetry. The particle content is necessarily increased to be free from anomalies, making it possible to incorporate the superfields for right-handed neutrinos and an SU(2)-singlet Higgs boson. The vacuum expectation value of this Higgs boson, which induces spontaneous breakdown of U(1) symmetry, yields large Majorana masses for the right-handed neutrinos, leading to small masses for the ordinary neutrinos. The linear coupling of SU(2)-doublet Higgs superfields, which is indispensable to the superpotential of the minimal supersymmetric standard model, is replaced by a trilinear coupling of the Higgs superfields, so that there is no mass parameter in the superpotential. The energy dependencies of the model parameters are studied, showing that gauge symmetry breaking is induced by radiative corrections. Certain ranges of the parameter values compatible with phenomena at the electroweak energy scale can be derived from universal values of masses-squared and trilinear coupling constants for scalar fields at a very high energy scale.

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I. INTRODUCTION

The standard model (SM) describes particle physics below the electroweak energy scale well. However, various theoretical considerations suggest that some extension of the SM may be necessary for physics above that energy scale. Various models therefore have been proposed, some of which have been studied extensively. Among them, extensions with supersymmetry [1] are considered most plausible. In particular, the minimal supersymmetric standard model (MSSM) is usually treated as the standard theory around the electroweak energy scale.

The MSSM inherits most of the successful features of the SM, while the extension being minimal. However, this model suffers one serious setback, which has been often passed over. In the SM, the proton is protected from decay naturally by gauge symmetry. On the other hand, in the MSSM, the gauge symmetry allows the interactions of dimension four which do not conserve baryon and/or lepton numbers. Unless there exists some reason to forbid these interactions, the proton decays in an unacceptably short time. Therefore, a discrete symmetry is usually imposed on the MSSM through R parity, which is however merely an *ad hoc* symmetry.

A convincing reason for proton stability could be provided by an extra gauge symmetry. Although such a symmetry around the electroweak energy scale is subjected to many phenomenological constraints, they still show room to allow a U(1) gauge symmetry. Several supersymmetric models with an extra U(1) symmetry therefore have been discussed

[2–6], aiming at natural explanation for a long lifetime of the proton. However, these models are accompanied by some arbitrariness in construction, which might reduce reliability of the reasoning for the proton stability.

In this paper, the supersymmetric extension of the SM with an extra U(1) gauge symmetry is studied within the framework of a model coupled to $N=1$ supergravity. In addition to the proton lifetime, the MSSM involves problems on the neutrino masses and the linear coupling of Higgs superfields noted later. Requiring a model to solve these problems consistently in a minimal extension, its particle contents and superpotential are determined rather uniquely [7]. In sizable ranges of the model parameters, the scalar potential appropriately gives a vacuum of $SU(3) \times U_{EM}(1)$ gauge symmetry below the electroweak energy scale. Phenomenological predictions of the model are compatible with experimental results. A typical mass scale of scalar particles is of order 1 TeV, which can account for the smallness of the electric dipole moments (EDMs) of the neutron and the electron, another problem of the MSSM.

The energy dependencies of the model parameters are also discussed by analyzing renormalization group equations (RGEs). Taking the masses-squared of scalar fields all positive at a high energy scale, those for some Higgs fields become sufficiently small at lower energy scales to induce the breakdowns of the extra U(1) and electroweak gauge symmetries. In the model coupled to $N=1$ supergravity the masses-squared and the trilinear coupling constants for scalar fields are considered to respectively have universal values at a very high energy scale. This scenario can be realized in this model.

In constructing the model, we take into account the problems of the neutrino masses and the Higgs linear coupling as well as that of the proton stability. The former is raised by

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non-vanishing masses of the neutrinos suggested from experiments for atmospheric and solar neutrinos, such as at the Super-Kamiokande [8]. The MSSM or the SM can have Yukawa couplings for neutrino Dirac masses, if right-handed neutrinos are naively included. However, these fields are inert for the transformations of the gauge groups and their existence is not prescribed by the model. Furthermore, the extreme lightness of the neutrinos may require some explanation. Although this lightness could be attributed to large Majorana masses of the right-handed neutrinos, their origin is not clarified.

The latter problem is posed by a mass parameter of the μ term, a linear coupling of the Higgs superfields in the superpotential of the MSSM [9], which is indispensable for correct breaking of electroweak gauge symmetry. This mass parameter μ should have a magnitude of order the electroweak energy scale. The other mass parameters in the model are traced back to supersymmetry-soft-breaking terms of the Lagrangian and thus related to the gravitino mass which may be of order the electroweak energy scale. On the other hand, the μ parameter is contained in the supersymmetric term and its magnitude may be given arbitrarily. It is natural that there is no mass parameter in the superpotential and the role of the μ term is assumed by another effective μ term.

These problems could also be solved by introducing an extra U(1) gauge symmetry. Imposing a new gauge symmetry yields chiral and trace anomalies within the particle contents of the MSSM. For canceling the anomalies, new superfields are necessarily incorporated, among which those for right-handed neutrinos and an SU(2)-singlet Higgs boson may be included. The extra U(1) symmetry could be broken above the electroweak energy scale by a vacuum expectation value (VEV) of this Higgs boson, which may provide a source of Majorana masses for the right-handed neutrinos and the effective μ parameter.

This paper is organized as follows. In Sec. II we construct a model, in which the proton is adequately stable, the ordinary neutrinos have non-vanishing but small masses, and the effective μ term is contained. In Sec. III the vacuum structure of the model is discussed, paying particular attention to experimental constraints on an extra neutral gauge boson. In Sec. IV the behavior of the model parameters for different energy scales is analyzed through the RGEs to examine the radiative breaking of gauge symmetry and the supersymmetry breaking by $N=1$ supergravity. Conclusions and discussions are given in Sec. V.

II. MODEL

Particle contents of the model are constrained by the requirements of proton stability, neutrino masses, and an effective μ term. We also keep the extension of the SM as minimal as possible. The neutrino masses necessitate superfields for right-handed neutrinos, which are denoted by N^c. For the effective μ term an SU(3)×SU(2)×U(1) singlet superfield S is included. In addition, new colored superfields K and K^c are necessary for canceling a chiral anomaly, as shown later. In Table I we list the left-handed chiral superfields contained in the model with their quantum numbers under SU(3),

TABLE I. Particle contents and their quantum numbers. $i = 1,2,3$; $j = 1, \ldots, n_j$; $k = 1, \ldots, n_k$; $l = 1, \ldots, n_l$.

	SU(3)	SU(2)	U(1)	U$'$(1)
Q^i	3	2	$\frac{1}{6}$	Q_Q
U^{ci}	3*	1	$-\frac{2}{3}$	Q_{U^c}
D^{ci}	3*	1	$\frac{1}{3}$	Q_{D^c}
L^i	1	2	$-\frac{1}{2}$	Q_L
N^{ci}	1	1	0	Q_{N^c}
E^{ci}	1	1	1	Q_{E^c}
H_1^j	1	2	$-\frac{1}{2}$	Q_{H_1}
H_2^j	1	2	$\frac{1}{2}$	Q_{H_2}
S^k	1	1	0	Q_S
K^l	3	1	Y_K	Q_K
K^{cl}	3*	1	$-Y_K$	Q_{K^c}

SU(2), U(1), and U$'$(1) gauge transformations. The extra U(1) gauge symmetry is denoted by U$'$(1), for which the charges of superfields are expressed as Q_Q, Q_{U^c}, etc. In order not to yield chiral and trace anomalies within the standard gauge symmetries, we assign opposite U(1) charges Y_K and $-Y_K$ to K and K^c. The index i ($i=1,2,3$) stands for the generation, while the indices j of H_1 and H_2, k of S, and l of K and K^c are for possible multiplication to be determined by cancellation of the anomalies.

The superpotential should contain the couplings H_1QD^c, H_2QU^c, H_1LE^c, and H_2LN^c to generate masses for quarks and leptons. The μ term can be replaced by the coupling SH_1H_2, provided that the scalar component of S has a non-vanishing VEV. The Dirac masses of the neutrinos may be comparable to those of the charged leptons, unless the Yukawa coupling constants are extremely small. However, the ordinary neutrino masses are suppressed by giving large Majorana masses to the right-handed neutrinos, which can be accomplished, without another new field, by including the coupling SN^cN^c. These couplings provide constraints on the U$'$(1) charges of the superfields:

$$Q_{H_1} + Q_Q + Q_{D^c} = 0, \tag{1}$$

$$Q_{H_2} + Q_Q + Q_{U^c} = 0, \tag{2}$$

$$Q_{H_1} + Q_L + Q_{E^c} = 0, \tag{3}$$

$$Q_{H_2} + Q_L + Q_{N^c} = 0, \tag{4}$$

$$Q_S + Q_{H_1} + Q_{H_2} = 0, \tag{5}$$

$$Q_S + Q_{N^c} + Q_{N^c} = 0. \tag{6}$$

If colored superfields are only those which correspond to the quarks of the SM, the $[SU(3)]^2U'(1)$ anomaly-free condition with Eqs. (1) and (2) gives the relation $Q_{H_1} + Q_{H_2} = 0$. The linear coupling H_1H_2 are not forbidden in the superpotential, and thus the model inevitably has a mass parameter of unknown origin. Therefore, new colored superfields

should be included to solve the problem of the μ term. Although there are various candidates for such superfields, according to the ''minimal'' postulate, we incorporate a pair of superfields in the fundamental representations of the SU(3) group, K and K^c. Then, their fermion components should have large masses from a phenomenological viewpoint, which is fulfilled by allowing the coupling SKK^c. This coupling leads to another constraint

$$Q_S + Q_K + Q_{K^c} = 0. \tag{7}$$

The $[\text{SU}(3)]^2 \text{U}'(1)$ anomaly-free condition and Eqs. (1), (2), (5), and (7) fix the number n_l of pairs for K and K^c at three, which agrees with the number of the generation for quarks and leptons.

The number n_j of pairs for H_1 and H_2 and the number n_k for S are determined by the freedom from chiral anomalies for $[\text{SU}(2)]^2 \text{U}'(1)$, $[\text{U}(1)]^2 \text{U}'(1)$ and a trace anomaly for $\text{U}'(1)$ with Eqs. (1)–(5),(7). These constraints are satisfied by either of the three sets of numbers and U(1) charge:

(A) $n_j = 4$, $n_k = 5$, $Y_K = 0$,

(B) $n_j = 3$, $n_k = 3$, $Y_K = \pm\dfrac{1}{3}$,

(C) $n_j = 2$, $n_k = 1$, $Y_K = \pm\dfrac{\sqrt{2}}{3}$.

However, the solution (A) does not satisfy the $\text{U}(1)[\text{U}'(1)]^2$ anomaly-free condition with Eq. (6). The solution (C) gives irrational U(1) charges for K and K^c. The solution (B) is free from the $\text{U}(1)[\text{U}'(1)]^2$ anomaly, and also satisfies the remaining $[\text{U}'(1)]^3$ anomaly-free condition. Therefore, a plausible solution is uniquely given by the set (B). The numbers n_j and n_k again become equal to the number of the generation.

A further constraint comes from the stable proton. The allowed value of Y_K for the U(1) charges of K and K^c is now either 1/3 or $-1/3$. However, the proton stability by gauge symmetry is only achieved for $Y_K = 1/3$ [4,5]. For $Y_K = -1/3$, the particle contents of one generation can be embedded in the fundamental representation of the E_6 group. Unless a discrete symmetry is imposed, the baryon and/or lepton numbers are violated by couplings of dimension four, such as $U^c D^c K^c$ and LQK^c, which induce an unacceptably fast decay of the proton. On the other hand, for $Y_K = 1/3$, allowed couplings of dimension four are only those which have already been taken into account, i.e., $H_1 QD^c$, $H_2 QU^c$, $H_1 LE^c$, $H_2 LN^c$, $SH_1 H_2$, $SN^c N^c$, and SKK^c. Baryon number is conserved while lepton number is not, which is sufficient for the proton stability. The lowest dimension couplings of baryon-number violation are given by the D terms of $QQU^{c*}E^{c*}$, $QQD^{c*}N^{c*}$, and $QU^{c*}D^{c*}L$, which are of dimension six.

Under all the anomaly-free conditions and Eqs. (1)–(7), the $\text{U}'(1)$ charges of the superfields are expressed in terms of two independent variables. All the superfields are triplicated, and the anomalies are canceled in each generation.

The generators Y' and Y of $\text{U}'(1)$ and U(1), respectively, are required to be orthogonal, $\text{Tr}[Y'Y] = 0$. Then, the $\text{U}'(1)$ charges of the superfields are determined up to a normalization factor. For definiteness, hereafter, the $\text{U}'(1)$ charges are normalized to the U(1) charges as $\text{Tr}[Y'^2] = \text{Tr}[Y^2]$, which are shown in Table II.

The superpotential which contains all the couplings consistent with gauge symmetry and renormalizability is given by

$$\begin{aligned}
W = {}& \eta_d^{ijk} H_1^i Q^j D^{ck} + \eta_u^{ijk} H_2^i Q^j U^{ck} \\
&+ \eta_e^{ijk} H_1^i L^j E^{ck} + \eta_\nu^{ijk} H_2^i L^j N^{ck} \\
&+ \lambda_N^{ijk} S^i N^{cj} N^{ck} + \lambda_H^{ijk} S^i H_1^j H_2^k + \lambda_K^{ijk} S^i K^j K^{ck},
\end{aligned} \tag{8}$$

where η_d, η_u, η_e, η_ν, λ_N, λ_H, and λ_K represent dimensionless constants. Contraction of group indices is understood. In the MSSM without the discrete symmetry through R parity, the couplings $D^c D^c U^c$, LQD^c, LLE^c, $H_1 H_1 E^c$, and LH_2 are allowed, leading to non-conservation of baryon and lepton numbers. Here, these couplings are forbidden by the $\text{U}'(1)$ gauge symmetry. The proton decay could only occur through the operators of dimension six, being suppressed at least by a huge mass to the second power. As long as this mass scale is not much smaller than the Planck mass, the proton becomes adequately stable. The couplings of the superpotential are all cubic, and there is no mass parameter.

We assume that supersymmetry is broken through the ordinary mechanism based on $N = 1$ supergravity. Supergravity is spontaneously broken in a hidden sector at the Planck mass scale, and then supersymmetry in an observable sector is broken softly. At lower energy scales, the Lagrangian of the observable sector consists of a supersymmetric part and a supersymmetry-soft-breaking part prescribed by gauge symmetry and superpotential. The soft-breaking part contains mass terms for gauge fermions, and trilinear couplings and mass terms for scalar bosons:

$$\begin{aligned}
\mathcal{L}_{SB} = {}& -\frac{1}{2}(\tilde{m}_3 \bar{\lambda}_3 \lambda_3 + \tilde{m}_2 \bar{\lambda}_2 \lambda_2 + \tilde{m}_1 \bar{\lambda}_1 \lambda_1 + \tilde{m}_1' \bar{\lambda}_1' \lambda_1') \\
&- m_{3/2}(A_d^{ijk} \eta_d^{ijk} H_1^i Q^j D^{ck} + A_u^{ijk} \eta_u^{ijk} H_2^i Q^j U^{ck} \\
&+ A_e^{ijk} \eta_e^{ijk} H_1^i L^j E^{ck} + A_\nu^{ijk} \eta_\nu^{ijk} H_2^i L^j N^{ck} \\
&+ B_N^{ijk} \lambda_N^{ijk} S^i N^{cj} N^{ck} + B_H^{ijk} \lambda_H^{ijk} S^i H_1^j H_2^k \\
&+ B_K^{ijk} \lambda_K^{ijk} S^i K^j K^{ck}) + \text{H.c.} - M_{Q^i}^2 |Q^i|^2 \\
&- M_{U^{ci}}^2 |U^{ci}|^2 - M_{D^{ci}}^2 |D^{ci}|^2 - M_{L^i}^2 |L^i|^2 \\
&- M_{N^{ci}}^2 |N^{ci}|^2 - M_{E^{ci}}^2 |E^{ci}|^2 - M_{H_1^i}^2 |H_1^i|^2 - M_{H_2^i}^2 |H_2^i|^2 \\
&- M_{S^i}^2 |S^i|^2 - M_{K^i}^2 |K^i|^2 - M_{K^{ci}}^2 |K^{ci}|^2.
\end{aligned} \tag{9}$$

Here λ_3, λ_2, λ_1, and λ_1' represent gauge fermions for SU(3), SU(2), U(1), and $\text{U}'(1)$, respectively. Scalar bosons are denoted by the same symbols as the corresponding superfields. With $m_{3/2}$ being the gravitino mass, the coefficients

TABLE II. U$'$(1) charges of the superfields. $i=1,2,3$.

Q^i	U^{ci}	D^{ci}	L^i	N^{ci}	E^{ci}
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{7}{12}$	$\frac{7}{12}$	$-\frac{5}{12}$	$\frac{1}{12}$
H_1^i	H_2^i	S^i	K^i	K^{ci}	
$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{1}{6}$	

A_d, A_u, A_e, A_ν, B_N, B_H, and B_K are dimensionless. At high energy scales not much lower than the Planck mass, the masses-squared of scalar fields are all around $m_{3/2}^2$ and positive. The trilinear coupling constants for scalar fields are also approximately the same. Around the electroweak energy scale, some of these parameters differ significantly from the high-energy values through large quantum corrections.

III. VACUUM STRUCTURE

The Lagrangian of our model has SU(3)×SU(2) ×U(1)×U$'$(1) gauge symmetry, which must be spontaneously broken down to SU(3)×U$_{EM}$(1) symmetry. This breaking could be achieved by VEVs for the scalar components of H_1^i, H_2^i, and S^i. We discuss the vacuum structure of the model by examining the scalar potential. Hereafter, we adopt the same notation for the superfields and their scalar components.

Although the scalar potential could contain all of H_1^i, H_2^i, and S^i and thus its general analysis is complicated, it may be simplified under certain assumptions. If the couplings between different generations are not significant, $H_2^3 Q^3 U^{c3}$ of the third generation has a large coefficient related to the mass of the top quark. The mass-squared of H_2^3 then receives large negative contributions through quantum corrections and becomes small around the electroweak energy scale. As a result, H_1^3 and H_2^3 can have nonvanishing VEVs and assume the role of two Higgs doublets in the MSSM. For the first two generations, on the other hand, such couplings for H_1^i or H_2^i have small coefficients, so that the masses-squared of these scalar fields are kept around $m_{3/2}^2$. If the coefficient of $S^3 K^3 K^{c3}$ is large, the mass-squared of S^3 is also driven small. Although there is no phenomenological information about $S^i K^i K^{ci}$, a hierarchy of their coefficients could well exist. Among the three scalar fields S^i, one scalar field S^3 alone may have a nonvanishing VEV. We thus assume that only H_1^3, H_2^3, and S^3 can have nonvanishing VEVs. Quantum corrections to the masses-squared of other scalar fields are small, keeping them around $m_{3/2}^2$. The masses-squared of K^i, K^{ci}, and N^{ci} receive non-negligible negative contributions from the D-term of U$'$(1) when the gauge symmetry is broken spontaneously. However, the positive contributions from the supersymmetry-soft-breaking terms in Eq. (9) can dominate over and prevent these scalar fields from getting nonvanishing VEVs.

Assuming the above simplification, the scalar potential is given by

$$V = \frac{1}{8}g_2^2(|H_1|^2+|H_2|^2)^2 + \frac{1}{8}g_1^2(|H_1|^2-|H_2|^2)^2$$

$$+ \frac{1}{72}g_1'^2(4|H_1|^2+|H_2|^2-5|S|^2)^2$$

$$- \left(\frac{1}{2}g_2^2-|\lambda_H|^2\right)|H_1 H_2|^2 + |\lambda_H|^2(|H_1|^2+|H_2|^2)|S|^2$$

$$+ (B_H \lambda_H m_{3/2} S H_1 H_2 + \text{H.c.}) + M_{H_1}^2|H_1|^2$$

$$+ M_{H_2}^2|H_2|^2 + M_S^2|S|^2, \tag{10}$$

where the generation indices are left out. With group indices being expressed, $H_1 H_2$ is written as $\epsilon_{ab}H_{1a}H_{2b}$, so that holds an equation $|H_1 H_2|^2 = |H_1|^2|H_2|^2 - |H_1^\dagger H_2|^2$. The gauge coupling constants for SU(2), U(1), and U$'$(1) are denoted by g_2, g_1, and g_1', respectively.

We now discuss the VEVs of the Higgs fields $\langle H_1 \rangle$, $\langle H_2 \rangle$, and $\langle S \rangle$. For any values of $\langle H_1 \rangle$ and $\langle H_2 \rangle$, the complex phase of $\langle S \rangle$ has a value which gives an equality $B_H \lambda_H m_{3/2}\langle SH_1 H_2\rangle = -|B_H \lambda_H m_{3/2}\langle SH_1 H_2\rangle|$. For given values of $\langle|H_1|^2\rangle$ and $\langle|H_2|^2\rangle$, the VEV $\langle|H_1 H_2|^2\rangle$ becomes maximum at $\langle H_1^\dagger H_2\rangle = 0$. Therefore, a condition $g_2^2 > 2|\lambda_H|^2$ guarantees electric charge conservation. Differently from the MSSM, there is no direction for the VEVs where their quartic terms are absent in the scalar potential. The potential gives a stable vacuum irrespectively of the supersymmetry-soft-breaking terms. Redefining the global phases of the Higgs fields so as to give $B_H \lambda_H = -|B_H \lambda_H|$, the VEVs v_1, v_2, and v_s of the neutral components of H_1, H_2, and S, respectively, may be taken real and non-negative. If these VEVs are all non-vanishing, extremum conditions are given by

$$\frac{1}{8}(g_2^2+g_1^2)(v_1^2-v_2^2)v_1 + \frac{1}{18}g_1'^2(4v_1^2+v_2^2-5v_s^2)v_1$$

$$+ \frac{1}{2}|\lambda_H|^2(v_2^2+v_s^2)v_1 - \frac{1}{\sqrt{2}}|B_H \lambda_H m_{3/2}|v_2 v_s + M_{H_1}^2 v_1$$

$$= 0, \tag{11}$$

$$-\frac{1}{8}(g_2^2+g_1^2)(v_1^2-v_2^2)v_2 + \frac{1}{72}g_1'^2(4v_1^2+v_2^2-5v_s^2)v_2$$

$$+ \frac{1}{2}|\lambda_H|^2(v_1^2+v_s^2)v_2 - \frac{1}{\sqrt{2}}|B_H \lambda_H m_{3/2}|v_1 v_s + M_{H_2}^2 v_2$$

$$= 0, \tag{12}$$

$$-\frac{5}{72}g_1'^2(4v_1^2+v_2^2-5v_s^2)v_s + \frac{1}{2}|\lambda_H|^2(v_1^2+v_2^2)v_s$$

$$- \frac{1}{\sqrt{2}}|B_H \lambda_H m_{3/2}|v_1 v_2 + M_S^2 v_s = 0. \tag{13}$$

It turns out that the solution of these simultaneous equations is unique, if it exists. The true vacuum is either at such a point or at a point where at least one VEV vanishes, being determined by the potential energies of those points.

The VEVs of the Higgs bosons have to satisfy phenomenological constraints coming from experiments for the gauge bosons. The W-boson mass has been measured precisely. The Z boson for $SU(2) \times U(1)$ and the Z' boson for $U'(1)$ are mixed and their mass-squared matrix is given by

$$\begin{pmatrix} M_Z^2 & M_{ZZ'}^2 \\ M_{ZZ'}^2 & M_{Z'}^2 \end{pmatrix}, \tag{14}$$

$$M_Z^2 = \frac{1}{4}(g_2^2 + g_1^2)(v_1^2 + v_2^2), \tag{15}$$

$$M_{Z'}^2 = \frac{1}{36} g_1'^2 (16 v_1^2 + v_2^2 + 25 v_s^2), \tag{16}$$

$$M_{ZZ'}^2 = \frac{1}{12} g_1' \sqrt{g_2^2 + g_1^2}(4 v_1^2 - v_2^2). \tag{17}$$

Two massive neutral gauge bosons, which are denoted by Z_1 and Z_2 ($M_{Z_1} < M_{Z_2}$), are predicted. The measured mass for the Z boson of the SM should be taken as the mass of Z_1. The experimental lower bound on the mass of a new neutral gauge boson is about 600 GeV [10]. According to detailed analyses of various experiments for an extra gauge boson [11], the mixing between Z and Z' is small. Defining a mixing parameter by $R = (M_{ZZ'}^2)^2 / M_Z^2 M_{Z'}^2$, a bound $R \lesssim 10^{-3}$ is roughly obtained. The VEVs can also be constrained by the lightest Higgs boson mass, whose experimental bound is given by $M_{H^0} \gtrsim 80$ GeV [12]. Since its predicted mass by the tree-level potential in Eq. (10) could be altered to become larger by several tens of GeV through one-loop quantum corrections, we conservatively put a constraint $M_{H^0} > 50$ GeV to the tree-level mass.

The scalar potential is analyzed numerically. For independent coefficients of the potential we choose $|\lambda_H|$, $|B_H \lambda_H m_{3/2}|$, $M_{H_1}^2$, $M_{H_2}^2$, and M_S^2. In Fig. 1 we show the regions for $M_{H_1}^2$ and $M_{H_2}^2$ where the VEVs are compatible with the above constraints. We have also imposed the constraints $1 \leq v_2/v_1 \leq 35$ and $M_{Z_2} \leq 2000$ GeV. With $|B_H \lambda_H m_{3/2}|$ being 0.1 TeV, $|\lambda_H|$ is set for 0.1 and 0.3, which correspond to the upper and lower regions, respectively. For given values of $M_{H_1}^2$, $M_{H_2}^2$, $|\lambda_H|$, and $|B_H \lambda_H m_{3/2}|$, the remaining parameter M_S^2 is so determined as to make the W-boson mass coincident with the measured value. The gauge coupling constant for $U'(1)$ is taken for $g_1' = g_1$. Owing to the constraints from M_{Z_2} and R, in wide regions $M_{H_1}^2$ is larger than $(1 \text{ TeV})^2$. The value of $M_{H_2}^2$ is generally smaller than $M_{H_1}^2$ in magnitude. The region for $|\lambda_H| = 0.3$ with $M_{H_1}^2 \lesssim (500 \text{ GeV})^2$ corresponds to $v_2/v_1 \lesssim 2$. A rough estimate of Eq. (12) shows that the sign of $M_{H_2}^2$ is positive for $|\lambda_H|^2 < (5/36) g_1'^2$ while $M_{H_2}^2$ has either sign for larger
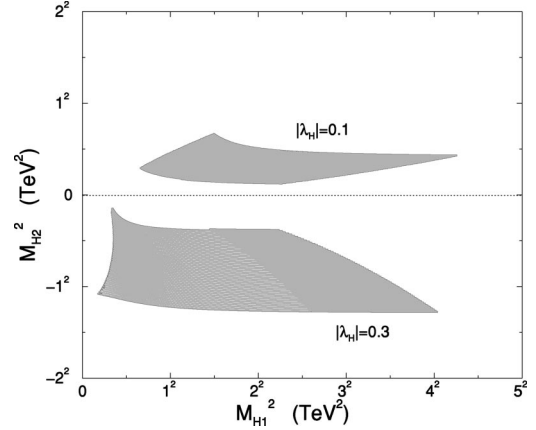

FIG. 1. The regions for $M_{H_1}^2$ and $M_{H_2}^2$ consistent with the experimental constraints. The other parameters are taken for $|B_H \lambda_H m_{3/2}| = 0.1$ TeV and $|\lambda_H| = 0.1, 0.3$.

values of $|\lambda_H|$. The value of M_S^2 is smaller than $M_{H_2}^2$ and always negative. As $|B_H \lambda_H m_{3/2}|$ increases, the allowed values for $M_{H_1}^2$ become larger, which is seen from Eq. (11). If the upper limit for M_{Z_2} is lifted, wider parameter regions become allowed. However, as the scale of the mass-squared parameters increases, more fine-tuning of the parameters becomes inevitable for electroweak symmetry breaking. For having the correct vacuum, large differences among $M_{H_1}^2$, $M_{H_2}^2$, and M_S^2 are necessary, which could well occur under our assumption for supersymmetry breaking.

In Table III we present four examples for the values of $M_{H_1}^2$, $M_{H_2}^2$, and M_S^2 in the allowed regions of Fig. 1. Also shown are the resultant values for v_2/v_1, v_s, M_{Z_2}, R, and the masses of the physical Higgs bosons. These Higgs-boson masses have been calculated, assuming for definiteness that the mass eigenstates are formed by the Higgs fields H_1, H_2, and S without mixing with the other fields of H_1^i, H_2^i, and S^i. Therefore, there are three mass eigenstates for the neutral

TABLE III. The parameter values of the potential and the outcomes. $|B_H \lambda_H m_{3/2}| = 0.1$ TeV.

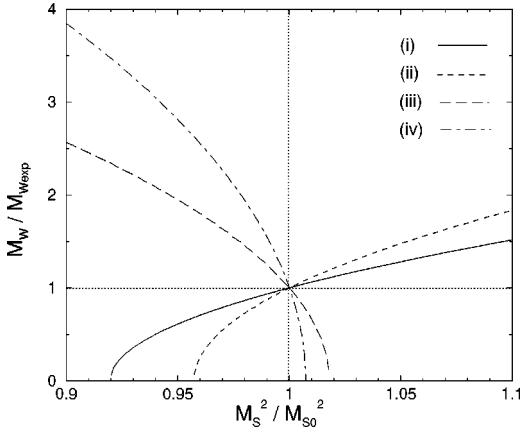|	(i)	(ii)	(iii)	(iv)
λ_H	0.1	0.1	0.3	0.3
$M_{H_1}^2$ (TeV^2)	1.0^2	2.0^2	1.0^2	2.0^2
$M_{H_2}^2$ (TeV^2)	0.20^2	0.30^2	-0.40^2	-0.60^2
M_S^2 (TeV^2)	-0.448^2	-0.888^2	-0.470^2	-0.671^2
v_2/v_1	5.7	11.5	6.7	18.2
v_s (TeV)	2.1	4.2	2.2	3.2
M_{Z_2} (TeV)	0.64	1.26	0.66	0.95
R	3.9×10^{-4}	1.3×10^{-4}	3.9×10^{-4}	2.4×10^{-4}
M_{H^0} (TeV)	0.086	0.091	0.074	0.072
	0.63	1.26	0.66	0.95
	0.94	1.87	1.04	2.03
M_{A^0} (TeV)	0.94	1.87	1.04	2.03
$M_{H^\pm}$ (TeV)	0.94	1.87	1.04	2.03

FIG. 2. The M_S^2 dependency of the predicted W-boson mass for the examples (i), (ii), (iii), and (iv) in Table III. M_{S0}^2 represents the values of M_S^2 which give the measured mass of the W boson M_{Wexp}.

scalar bosons H^0, one for the neutral pseudoscalar boson A^0, and one for the charged scalar boson $H^\pm$. One neutral scalar boson is light, whereas the others have large masses. The mixing parameter R vanishes for $v_2/v_1 = 2$, as seen from Eq. (17).

The large mass difference between Z_1 and Z_2 requires in some degree fine-tuning for the parameters in the potential. Since these two masses are different from each other by one order of magnitude, it is generally necessary to adjust the values of the mass-squared parameters $M_{H_1}^2$, $M_{H_2}^2$, M_S^2, and the coupling constants λ_H, B_H within the accuracy of order 10^{-2}. In Fig. 2, for the four examples in Table III, the ratio of a predicted W-boson mass to the experimental value is depicted as a function of M_S^2 normalized to its proper value which yields the correct W-boson mass. If the value of one parameter alone in the potential is deviated by order of 10^{-1}, the resultant VEVs may lead to a W-boson mass a few times its experimental value.

The neutrinos have both Dirac and Majorana masses. Neglecting the generation mixing, the mass matrix for the left-handed and right-handed neutrinos becomes

$$
\begin{pmatrix}
0 & -\eta_\nu v_2/\sqrt{2} \\
-\eta_\nu v_2/\sqrt{2} & \sqrt{2}\lambda_N v_s
\end{pmatrix},
\tag{18}
$$

whose lighter mass eigenvalue is approximately given by $m_{\nu 1} = |\eta_\nu|^2 v_2^2/2\sqrt{2}|\lambda_N|v_s$. With $|\lambda_N| = 0.2$ and $v_s = 3$ TeV, $m_{\nu 1}$ becomes about 59 eV for $|\eta_\nu|v_2 = 10$ MeV and 0.59 eV for $|\eta_\nu|v_2 = 1$ MeV. Even if the Yukawa coupling constants for the neutrino Dirac masses are of the same order as that for the electron, the observed ordinary neutrinos have tiny masses which could be a reason for the recent experimental results suggesting neutrino oscillations.

The coupling SH_1H_2 serves as the μ term and an effective μ parameter is given by $\mu = \lambda_H v_s/\sqrt{2}$. For $|\lambda_H| = 0.2$ and $v_s = 3$ TeV, $|\mu|$ is approximately 420 GeV. The parameter μ can have an appropriate magnitude for the electroweak symmetry breaking. This parameter also affects the

masses of the charginos and the neutralinos. Assuming that these particles are formed by the fermion components of H_1, H_2, and S, as well as the SU(2), U(1), and U$'(1)$ gauge fermions, there exist two mass eigenstates for the charginos and six mass eigenstates for the neutralinos. Provided that the gauge fermions for SU(2), U(1), and U$'(1)$ receive masses of order 100 GeV from the supersymmetry-soft-breaking terms in Eq. (9), the masses of the lighter chargino and the lightest neutralino become of order 100 GeV.

This model contains new particles which are not predicted by the MSSM. As already noted, the gauge-Higgs sector involves extra a neutral gauge boson, a neutral Higgs boson, and two neutralinos. For the lepton sector, there appears a heavy neutrino in each generation. Correspondingly the scalar neutrinos are duplicated. The interactions arising from the coupling SN^cN^c do not conserve lepton number. In addition, the superfields H_1^i, H_2^i, S^i with $i=1,2$ and K^j, K^{cj} with $j=1,2,3$ are newly introduced. The masses of their fermion components are generated by the couplings to H_1^3, H_2^3, and S^3, and become of order $0.1-1$ TeV. The lightest fermion among K^j and K^{cj} is stable. As well as by collider experiments, such a stable particle may be explored by other methods to search for its relics in the universe, e.g. anomalous nuclei in sea water. However, these methods depend on the relic density, whose theoretical prediction is plagued by various uncertainties for non-perturbative effects, cosmology, and so on. Since the scalar components of H_1^i and H_2^i couple to quarks and leptons, non-trivial constraints are imposed on their coupling constants from the viewpoint of flavor-changing neutral current. However, these scalar particles are rather heavy, so that the constraints are not so stringent as usually thought. If the couplings $S^iH_1^3H_2^3$, $S^3H_1^iH_2^3$, or $S^3H_1^3H_2^i$ are not neglected, some or all of the scalar or fermion components for H_1^i, H_2^i, and S^i are mixed with the Higgs bosons, charginos, or neutralinos, leading to an enlargement of the particles belonging to the gauge-Higgs sector.

The MSSM has another problem on the EDMs of the neutron and the electron, which can be explained in this model. If the squark and slepton masses are of order 100 GeV and the CP-violating phases intrinsic in the model are not suppressed, these EDMs are predicted to be much larger than their experimental upper bounds. However, a typical scale of the squark and slepton masses in this model, which are considered not much different from the mass of H_1, is larger than 1 TeV. The EDMs then lie within the experimental bounds without fine-tuning the CP-violating phases to be very small [13]. If these phases are not suppressed, the interactions of the charginos or the neutralinos generally induce sizable CP violation in their production or decay processes.

IV. ENERGY DEPENDENCE

The parameter values of the model change according to the relevant energy scale. Analyzing their energy dependencies, we discuss whether gauge symmetry breaking is induced by radiative corrections. We also examine the scenario of universal values for the masses-squared and the trilinear
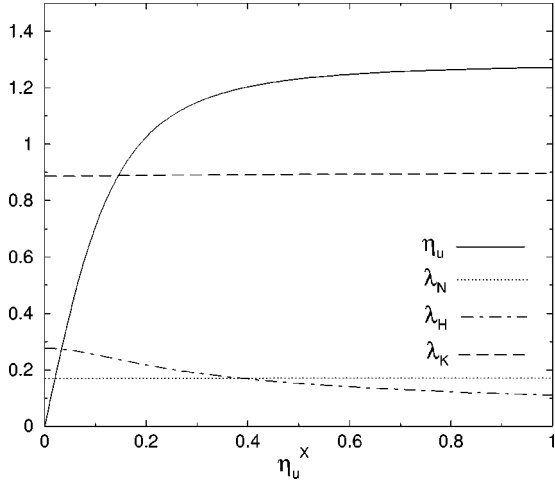
FIG. 3. The low-energy values of η_u, λ_N, λ_H, λ_K for $\eta_u^X = 0$ -1 and $\lambda_N^X = \lambda_H^X = \lambda_K^X = 0.2$.

coupling constants of scalar fields at a very high energy scale. For simplicity, the generation mixing of the particle fields are neglected.

The evolution of the parameters concerning the energy-scale change are described by RGEs, which are given in the Appendix. It is seen from those equations that $M_{H_2}^2$ increases as the energy scale becomes high, owing to a large Yukawa coupling constant η_u for the top quark. If the coupling constant λ_K is around unity, M_S^2 also increases. Consequently, the mass-squared parameters can all have large positive values at high energy scales, even if they are small at a low energy scale as discussed in the previous section. The $SU(2) \times U(1) \times U'(1)$ symmetry is spontaneously broken through radiative corrections. The experimental values of the gauge coupling constants suggest that these constants are not unified at the energy scale for possible grand unification. This gauge unification could be achieved by incorporating one additional pair of SU(2)-doublet chiral superfields. However, such a pair form a gauge-singlet linear coupling and thus ruin the model by necessitating a mass parameter of unknown origin. Although the particle contents are not embedded in the fundamental representation of the E_6 group, the masses and the coupling constants evolve similarly to those in the E_6 models. Some features of these models [14] apply to the present model, and vice versa.

We now numerically examine the evolution of the parameters. Taking the masses-squared and the trilinear coupling constants of the scalar fields for common values $m_{3/2}^2$ and A at a high energy scale M_X, we evaluate the parameters at a low energy scale M. For definiteness, we set M_X for 10^{17} GeV and M for 5×10^2 GeV. Assuming an equality $g_1 = g_1'$, all the gauge coupling constants are determined independently of the parameters. The masses of the gauge fermions are also determined, if their values are given at some energy scale. Since the gauge groups are not unified in our model, these masses at M_X are generally different from each other. However, they are considered nevertheless to be of the same order of magnitude. We therefore put their values equal at M_X, $\tilde{m}_3 = \tilde{m}_2 = \tilde{m}_1 = \tilde{m}_1' \equiv \tilde{m}$, for simplicity. The Yukawa
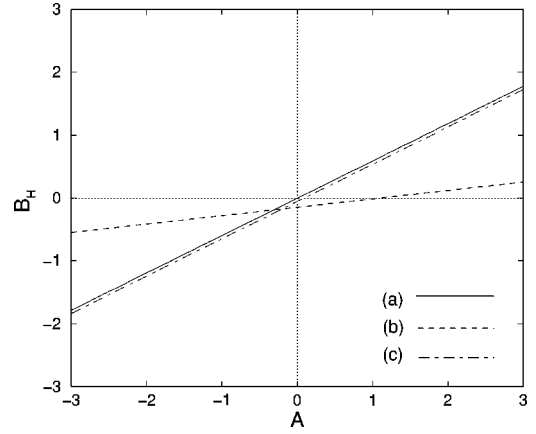
coupling constants η_u, η_d, η_ν, η_e, λ_N, λ_H, and λ_K at M_X, which are specified by attaching an index ''X,'' are independent of each other.

In Fig. 3 the values of η_u, λ_N, λ_H, and λ_K at M are depicted as functions of η_u^X. We have taken $\lambda_N^X = \lambda_H^X = \lambda_K^X = 0.2$ and $\eta_d^X = \eta_\nu^X = \eta_e^X = 0$. The magnitude of η_u and λ_K become large at the low energy scale, while the energy dependencies of λ_N and λ_H are not significant. The evolution of each Yukawa coupling constant is not affected much by the other parameters. For generating an appropriate mass for the top quark, $|\eta_u^X|$ should be larger than 0.1. The condition $g_2^2 > 2|\lambda_H|^2$ for electric charge conservation at the low energy scale is satisfied for $|\lambda_H^X| \lesssim 0.5$.

In Fig. 4 we show the trilinear coupling constant B_H as a function of A for three sets of η_u^X, λ_K^X, and $\tilde{m}/m_{3/2}$ given in Table IV. For the other non-vanishing input parameters, we take $\lambda_N^X = \lambda_H^X = 0.2$. The magnitude of A is constrained as $|A| < 3$ in order not to induce incorrect breaking of gauge symmetry. The value of B_H is of order unity in wide ranges of A except narrow ranges where $|B_H|$ is much smaller than unity. The $\tilde{m}$ dependence of B_H is negligible.

The mass-squared parameters $M_{H_1}^2$, $M_{H_2}^2$, and M_S^2 are shown, as functions of A, for the three parameter sets (a), (b), and (c) of Table IV in Figs. 5(a), 5(b), and 5(c), respectively, with $\lambda_N^X = \lambda_H^X = 0.2$. The gravitino mass is fixed as $m_{3/2} = 2$ TeV. At the low energy scale, receiving large quantum corrections, $M_{H_2}^2$ and M_S^2 could become much smaller than the universal value $m_{3/2}^2$. These corrections strongly depend on η_u^X, λ_K^X, $\tilde{m}$, and A. In Fig. 5(a), $M_{H_2}^2$ and M_S^2 become negative only for $|A| \gtrsim 2$. However, as η_u^X and λ_K^X increase, $M_{H_2}^2$

FIG. 4. The low-energy value of B_H for the parameter sets (a), (b), and (c) in Table IV. $\lambda_N^X = \lambda_H^X = 0.2$.

TABLE IV. The values of η_u^X, λ_K^X, and $\tilde{m}/m_{3/2}$ for numerical evaluations.

	(a)	(b)	(c)
η_u^X, λ_K^X	0.1	0.3	0.1
$\tilde{m}/m_{3/2}$	0.1	0.1	1

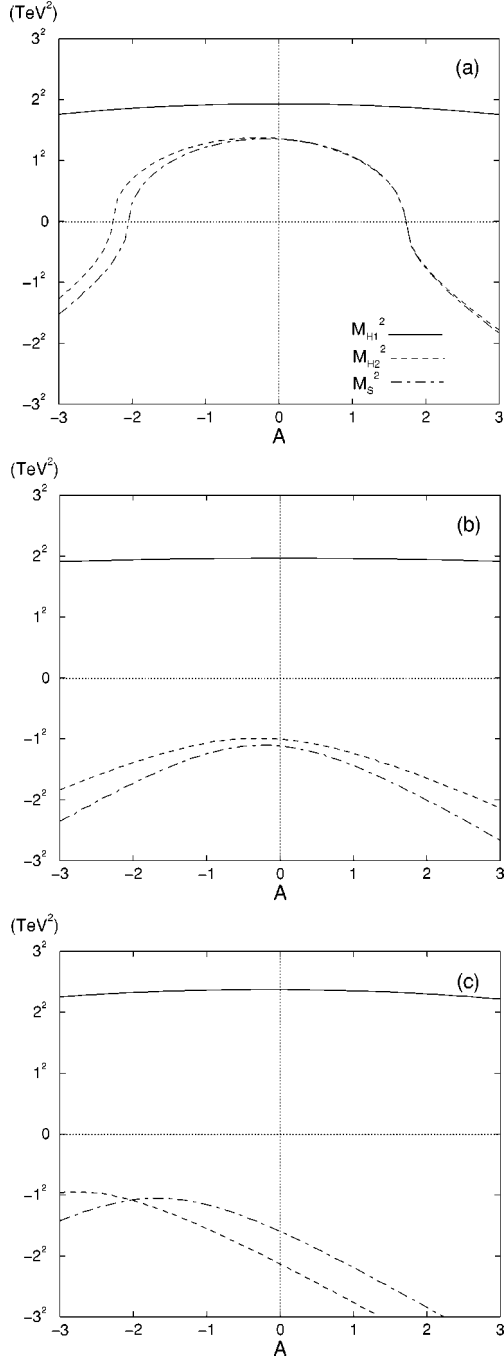FIG. 5. The low-energy values of $M_{H_1}^2$, $M_{H_2}^2$, and M_S^2 for the parameter sets (a), (b), and (c) in Table IV. $\lambda_N^X = \lambda_H^X = 0.2$, $m_{3/2} = 2$ TeV.

and M_S^2 decrease, respectively. These masses-squared also become small for larger values of $\tilde{m}$. In Figs. 5(b) and 5(c), $M_{H_2}^2$ and M_S^2 are negative for any value of A. If λ_K^X is larger than η_u^X, an inequality $M_S^2 < M_{H_2}^2$ holds at the low energy scale. On the other hand, $M_{H_1}^2$ is not much different from $m_{3/2}^2$.

We see from Figs. 3, 4, and 5 that the gauge symmetry of the vacuum at high energy scales can be spontaneously broken at low energy scales through radiative corrections. Fur-

TABLE V. The parameter values at the high energy scale and the low energy outcomes. $A = -1$, $\eta_u^X = \lambda_N^X = \lambda_K^X = 0.2$.

	(i)	(ii)	(iii)	(iv)
$m_{3/2}$ (TeV)	1.0	1.0	2.0	2.0
$\tilde{m}$ (TeV)	0.3	0.5	0.3	0.5
η_d^X	0.007	0.002	0.010	0.005
λ_H^X	0.356	0.412	0.307	0.339
v_2/v_1	3.2	2.1	5.2	3.7
v_s (TeV)	2.4	3.5	3.8	4.4
M_{Z_2} (TeV)	0.73	1.05	1.14	1.32
R	1.2×10^{-4}	2.1×10^{-6}	9.7×10^{-5}	5.3×10^{-5}
M_{H^0} (TeV)	0.062	0.042	0.068	0.066
	0.73	1.05	1.15	1.33
	1.06	1.32	1.95	2.05
M_{A^0} (TeV)	1.06	1.32	1.95	2.05
$M_{H^\pm}$ (TeV)	1.06	1.32	1.95	2.05
M_W (GeV)	80	82	74	79
m_t (GeV)	166	161	160	166
m_b (GeV)	2.9	1.2	2.4	1.8

thermore, certain parameter values at a high energy scale, with the masses-squared and the trilinear coupling constants of scalar fields being universal, lead to the values of $M_{H_1}^2$, $M_{H_2}^2$, M_S^2, λ_H, and B_H which give a plausible vacuum around the electroweak energy scale. As explicit examples, we show in Table V the parameter values at M_X which give low-energy vacua consistent with experimental results. Since there are many input parameters, for simplicity, we have fixed the trilinear coupling constant and the Yukawa coupling constants as $A = -1$ and $\eta_u^X = \lambda_N^X = \lambda_K^X = 0.2$. The values of $m_{3/2}$ and $\tilde{m}$ are set for $m_{3/2} = 1, 2$ TeV and $\tilde{m} = 0.3, 0.5$ TeV. The resultant values of v_2/v_1, v_s, M_{Z_2}, R, and the Higgs boson masses at M are also given together with the masses of the W boson M_W, the top quark m_t, and the bottom quark m_b. This model is compatible with the supersymmetry breaking mechanism based on $N = 1$ supergravity.

V. CONCLUSIONS

The MSSM involves the problems of proton lifetime, neutrino masses, and the μ term. These problems may be solved by theories at very high energy scales, which however are mere conjectures and mostly untestable at present. On the contrary, the solutions of the problems may reside in theories around an energy scale of the MSSM or the SM. Then, a new model beyond the MSSM should exist. Such a model is rigidly constructed on solid ground of various experimental results. Its predictions can be examined at experiments in the near future.

To solve the problems of the MSSM without invoking uncertain theories, we have constructed a supersymmetric standard model based on $SU(3) \times SU(2) \times U(1) \times U'(1)$ gauge symmetry and $N = 1$ supergravity. In this model, the solutions are given consistently within the framework of the model. The interactions of dimension four or five which vio-

late baryon number conservation are not allowed by the gauge symmetry, leading to an adequately long lifetime of the proton. The gauge symmetry also prescribes the existence of right-handed neutrinos and an SU(2)-singlet Higgs boson whose VEV v_s induces breaking of the U$'(1)$ symmetry. The neutrino masses receive contributions of Dirac type and those of Majorana type generated by the large VEV v_s. The ordinary neutrinos then have tiny masses. The μ term of the MSSM is replaced by a trilinear coupling of the Higgs superfields, and the effective μ parameter is given by v_s. A typical mass scale of this model is of order 1 TeV. Some fine-tuning of the parameters is necessary for correct breaking of the electroweak gauge symmetry. On the other hand, the EDMs of the neutron and the electron are predicted within their experimental bounds without fine-tuning much CP-violating phases.

The constructed model gives predictions different from the MSSM in various phenomenological aspects. An extra neutral gauge boson couples to all the quarks and leptons. There exists a stable fermion which is nontrivially transformed under SU(3) and U$_{EM}(1)$. Lepton number is not conserved in the interactions of the neutrinos or the scalar neu-

trinos. Some scalar particles mediate flavor-changing neutral current at the tree level. The experimental examination of these predictions could be performed in the near future.

Implications of this model for theories at higher energy scales have also been studied. The gauge symmetry breaking is induced by radiative corrections. The masses-squared and the trilinear coupling constants of scalar fields could be universal at an energy scale not much smaller than the Planck mass, which is consistent with the mechanism of supersymmetry breaking based on $N=1$ supergravity. The gauge coupling constants are not unified below the Planck mass scale, unless the particle contents are modified.

APPENDIX

The RGEs are listed below. The gauge and Yukawa coupling constants are expressed by $\alpha \equiv g^2/4\pi$ and $E \equiv \eta^2/4\pi$, $L \equiv \lambda^2/4\pi$.

The gauge coupling constants and the gauge fermion masses:

$$\mu \frac{d}{d\mu} \alpha_a = \frac{b_a}{2\pi} \alpha_a^2, \tag{A1}$$

$$\mu \frac{d}{d\mu} \tilde{m}_a = \frac{b_a}{2\pi} \alpha_a \tilde{m}_a, \tag{A2}$$

where $b_3 = 0$, $b_2 = 3$, $b_1 = 15$, and $b_1' = 15$ for SU(3), SU(2), U(1), and U$'(1)$, respectively.

The masses-squared of the scalar fields:

$$\mu \frac{d}{d\mu} M_Q^2 = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3\tilde{m}_3^2 + \frac{3}{4}\alpha_2\tilde{m}_2^2 + \frac{1}{36}\alpha_1\tilde{m}_1^2 + \frac{1}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\frac{1}{6}\alpha_1\xi + \frac{1}{12}\alpha_1'\xi'\right)$$
$$+ \frac{1}{2\pi}E_u(|A_u|^2 m_{3/2}^2 + M_{H_2}^2 + M_Q^2 + M_{U^c}^2) + \frac{1}{2\pi}E_d(|A_d|^2 m_{3/2}^2 + M_{H_1}^2 + M_Q^2 + M_{D^c}^2), \tag{A3}$$

$$\mu \frac{d}{d\mu} M_{U^c}^2 = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3\tilde{m}_3^2 + \frac{4}{9}\alpha_1\tilde{m}_1^2 + \frac{1}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(-\frac{2}{3}\alpha_1\xi + \frac{1}{12}\alpha_1'\xi'\right) + \frac{1}{\pi}E_u(|A_u|^2 m_{3/2}^2 + M_{H_2}^2 + M_Q^2 + M_{U^c}^2), \tag{A4}$$

$$\mu \frac{d}{d\mu} M_{D^c}^2 = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3\tilde{m}_3^2 + \frac{1}{9}\alpha_1\tilde{m}_1^2 + \frac{49}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\frac{1}{3}\alpha_1\xi + \frac{7}{12}\alpha_1'\xi'\right) + \frac{1}{\pi}E_d(|A_d|^2 m_{3/2}^2 + M_{H_1}^2 + M_Q^2 + M_{D^c}^2), \tag{A5}$$

$$\mu \frac{d}{d\mu} M_L^2 = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2\tilde{m}_2^2 + \frac{1}{4}\alpha_1\tilde{m}_1^2 + \frac{49}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(-\frac{1}{2}\alpha_1\xi + \frac{7}{12}\alpha_1'\xi'\right) + \frac{1}{2\pi}E_\nu(|A_\nu|^2 m_{3/2}^2 + M_{H_2}^2 + M_L^2 + M_{N^c}^2)$$
$$+ \frac{1}{2\pi}E_e(|A_e|^2 m_{3/2}^2 + M_{H_1}^2 + M_L^2 + M_{E^c}^2), \tag{A6}$$

$$\mu \frac{d}{d\mu} M_{N^c}^2 = -\frac{2}{\pi}\left(\frac{25}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(-\frac{5}{12}\alpha_1'\xi'\right) + \frac{1}{\pi}E_\nu(|A_\nu|^2 m_{3/2}^2 + M_{H_2}^2 + M_L^2 + M_{N^c}^2) + \frac{2}{\pi}L_N(|B_N|^2 m_{3/2}^2 + M_S^2 + 2M_{N^c}^2),$$
$$\text{(A7)}$$

$$\mu \frac{d}{d\mu} M_{E^c}^2 = -\frac{2}{\pi}\left(\alpha_1\tilde{m}_1^2 + \frac{1}{144}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\alpha_1\xi + \frac{1}{12}\alpha_1'\xi'\right) + \frac{1}{\pi}E_e(|A_e|^2 m_{3/2}^2 + M_{H_1}^2 + M_L^2 + M_{E^c}^2),$$
$$\text{(A8)}$$

$$\mu \frac{d}{d\mu} M_{H_1}^2 = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2\tilde{m}_2^2 + \frac{1}{4}\alpha_1\tilde{m}_1^2 + \frac{4}{9}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(-\frac{1}{2}\alpha_1\xi - \frac{2}{3}\alpha_1'\xi'\right) + \frac{3}{2\pi}E_d(|A_d|^2 m_{3/2}^2 + M_{H_1}^2 + M_Q^2 + M_{D^c}^2)$$

$$+ \frac{1}{2\pi}E_e(|A_e|^2 m_{3/2}^2 + M_{H_1}^2 + M_L^2 + M_{E^c}^2) + \frac{1}{2\pi}L_H(|B_H|^2 m_{3/2}^2 + M_S^2 + M_{H_1}^2 + M_{H_2}^2),$$
$$\text{(A9)}$$

$$\mu \frac{d}{d\mu} M_{H_2}^2 = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2\tilde{m}_2^2 + \frac{1}{4}\alpha_1\tilde{m}_1^2 + \frac{1}{36}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\frac{1}{2}\alpha_1\xi - \frac{1}{6}\alpha_1'\xi'\right) + \frac{3}{2\pi}E_u(|A_u|^2 m_{3/2}^2 + M_{H_2}^2 + M_Q^2 + M_{U^c}^2)$$

$$+ \frac{1}{2\pi}E_\nu(|A_\nu|^2 m_{3/2}^2 + M_{H_2}^2 + M_L^2 + M_{N^c}^2) + \frac{1}{2\pi}L_H(|B_H|^2 m_{3/2}^2 + M_S^2 + M_{H_1}^2 + M_{H_2}^2),$$
$$\text{(A10)}$$

$$\mu \frac{d}{d\mu} M_S^2 = -\frac{2}{\pi}\left(\frac{25}{36}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\frac{5}{6}\alpha_1'\xi'\right) + \frac{1}{\pi}L_N(|B_N|^2 m_{3/2}^2 + M_S^2 + 2M_{N^c}^2) + \frac{1}{\pi}L_H(|B_H|^2 m_{3/2}^2 + M_S^2 + M_{H_1}^2 + M_{H_2}^2)$$

$$+ \frac{3}{2\pi}L_K(|B_K|^2 m_{3/2}^2 + M_S^2 + M_K^2 + M_{K^c}^2),$$
$$\text{(A11)}$$

$$\mu \frac{d}{d\mu} M_K^2 = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3\tilde{m}_3^2 + \frac{1}{9}\alpha_1\tilde{m}_1^2 + \frac{4}{9}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(\frac{1}{3}\alpha_1\xi - \frac{2}{3}\alpha_1'\xi'\right) + \frac{1}{2\pi}L_K(|B_K|^2 m_{3/2}^2 + M_S^2 + M_K^2 + M_{K^c}^2),$$
$$\text{(A12)}$$

$$\mu \frac{d}{d\mu} M_{K^c}^2 = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3\tilde{m}_3^2 + \frac{1}{9}\alpha_1\tilde{m}_1^2 + \frac{1}{36}\alpha_1'\tilde{m}_1'^2\right) + \frac{1}{2\pi}\left(-\frac{1}{3}\alpha_1\xi - \frac{1}{6}\alpha_1'\xi'\right) + \frac{1}{2\pi}L_K(|B_K|^2 m_{3/2}^2 + M_S^2 + M_K^2 + M_{K^c}^2),$$
$$\text{(A13)}$$

where $\xi = \Sigma Y_\phi M_\phi^2$ and $\xi' = \Sigma Q_\phi M_\phi^2$.

The Yukawa coupling constants:

$$\mu \frac{d}{d\mu} E_u = -\frac{2}{\pi}E_u\left\{\frac{4}{3}\alpha_3 + \frac{3}{4}\alpha_2 + \frac{13}{36}\alpha_1 + \frac{1}{48}\alpha_1' - \frac{1}{4}(6E_u + E_d + E_\nu + L_H)\right\},$$
$$\text{(A14)}$$

$$\mu \frac{d}{d\mu} E_d = -\frac{2}{\pi}E_d\left\{\frac{4}{3}\alpha_3 + \frac{3}{4}\alpha_2 + \frac{7}{36}\alpha_1 + \frac{19}{48}\alpha_1' - \frac{1}{4}(E_u + 6E_d + E_e + L_H)\right\},$$
$$\text{(A15)}$$

$$\mu \frac{d}{d\mu} E_\nu = -\frac{2}{\pi}E_\nu\left\{\frac{3}{4}\alpha_2 + \frac{1}{4}\alpha_1 + \frac{13}{48}\alpha_1' - \frac{1}{4}(3E_u + 4E_\nu + E_e + 4L_N + L_H)\right\},$$
$$\text{(A16)}$$

$$\mu \frac{d}{d\mu} E_e = -\frac{2}{\pi}E_e\left\{\frac{3}{4}\alpha_2 + \frac{3}{4}\alpha_1 + \frac{19}{48}\alpha_1' - \frac{1}{4}(3E_d + E_\nu + 4E_e + L_H)\right\},$$
$$\text{(A17)}$$

$$\mu \frac{d}{d\mu} L_N = -\frac{2}{\pi}L_N\left\{\frac{25}{48}\alpha_1' - \frac{1}{4}(4E_\nu + 10L_N + 2L_H + 3L_K)\right\},$$
$$\text{(A18)}$$

$$\mu \frac{d}{d\mu} L_H = -\frac{2}{\pi}L_H\left\{\frac{3}{4}\alpha_2 + \frac{1}{4}\alpha_1 + \frac{7}{12}\alpha_1' - \frac{1}{4}(3E_u + 3E_d + E_\nu + E_e + 2L_N + 4L_H + 3L_K)\right\},$$
$$\text{(A19)}$$

$$\mu \frac{d}{d\mu} L_K = -\frac{2}{\pi}L_K\left\{\frac{4}{3}\alpha_3 + \frac{1}{9}\alpha_1 + \frac{7}{12}\alpha_1' - \frac{1}{4}(2L_N + 2L_H + 5L_K)\right\}.$$
$$\text{(A20)}$$

The trilinear coupling constants:

$$\mu \frac{d}{d\mu} A_u = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3 \tilde{m}_3 + \frac{3}{4}\alpha_2 \tilde{m}_2 + \frac{13}{36}\alpha_1 \tilde{m}_1 + \frac{1}{48}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(6A_u E_u + A_d E_d + A_\nu E_\nu + B_H L_H), \tag{A21}$$

$$\mu \frac{d}{d\mu} A_d = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3 \tilde{m}_3 + \frac{3}{4}\alpha_2 \tilde{m}_2 + \frac{7}{36}\alpha_1 \tilde{m}_1 + \frac{19}{48}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(A_u E_u + 6A_d E_d + A_e E_e + B_H L_H), \tag{A22}$$

$$\mu \frac{d}{d\mu} A_\nu = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2 \tilde{m}_2 + \frac{1}{4}\alpha_1 \tilde{m}_1 + \frac{13}{48}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(3A_u E_u + 4A_\nu E_\nu + A_e E_e + 4B_N L_N + B_H L_H), \tag{A23}$$

$$\mu \frac{d}{d\mu} A_e = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2 \tilde{m}_2 + \frac{3}{4}\alpha_1 \tilde{m}_1 + \frac{19}{48}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(3A_d E_d + A_\nu E_\nu + 4A_e E_e + B_H L_H), \tag{A24}$$

$$\mu \frac{d}{d\mu} B_N = -\frac{2}{\pi}\left(\frac{25}{48}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(4A_\nu E_\nu + 10B_N L_N + 2B_H L_H + 3B_K L_K), \tag{A25}$$

$$\mu \frac{d}{d\mu} B_H = -\frac{2}{\pi}\left(\frac{3}{4}\alpha_2 \tilde{m}_2 + \frac{1}{4}\alpha_1 \tilde{m}_1 + \frac{7}{12}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(3A_u E_u + 3A_d E_d + A_\nu E_\nu + A_e E_e + 2B_N L_N + 4B_H L_H + 3B_K L_K), \tag{A26}$$

$$\mu \frac{d}{d\mu} B_K = -\frac{2}{\pi}\left(\frac{4}{3}\alpha_3 \tilde{m}_3 + \frac{1}{9}\alpha_1 \tilde{m}_1 + \frac{7}{12}\alpha'_1 \tilde{m}'_1\right)\frac{1}{m_{3/2}} + \frac{1}{2\pi}(2B_N L_N + 2B_H L_H + 5B_K L_K). \tag{A27}$$

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